SOLUTIONS

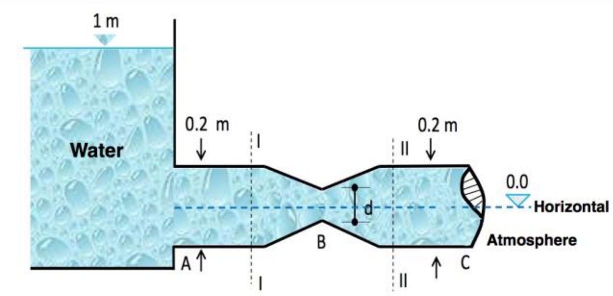
Question 1: There exists an incompressible, ideal and permanent (steady) flow of water in the reservoir-pipe system as shown in the figure given below. Water is poured into the atmosphere from a horizontal pipe ABC. Taking the absolute atmospheric pressure as 9.81 N/cm^2 and absolute vapor pressure as 0.23 N/cm^2:

a- Calculate the discharge of the system.

b- Without changing the discharge and letting the water evaporate, find the possible minimum value for the diameter of pipe B.

c- Draw the hydraulic and energy grade lines of the system.

d- Find the force that the flow exerts on the narrowing and expanding sections of the pipe choosing the control volume between cross-sections (I-I) and (II-II).



Solution 1:

If we write the BERNOULLI equation between O and C,

$$Z_0 + \frac{p_0}{\gamma} + \frac{v_0^2}{2g} = Z_E + \frac{p_E}{\gamma} + \frac{v_E^2}{2g}$$

$$1 + 0 + 0 = 0 + 0 + \frac{v_E^2}{2g} \rightarrow v_E^2 = 2g \rightarrow v_E = \sqrt{19.62} \rightarrow v_E = 4.42\, m/s$$

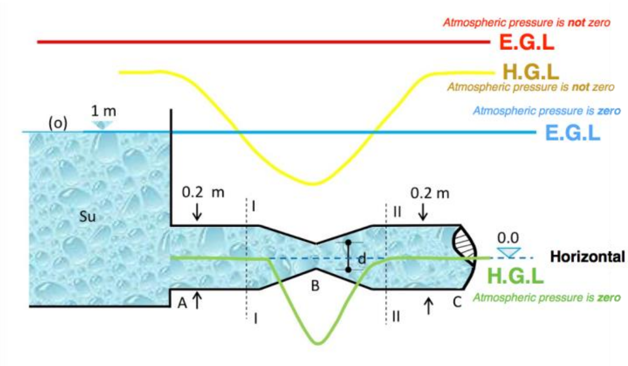
$$Q = v_E x A_E \rightarrow Q = 4.42x\frac{(0.2)^2\pi}{4} \rightarrow Q = 0.14\, m^3/s$$

If we write the BERNOULLI equation between B and C,

$$Z_B + \frac{(p_B)_m}{\gamma} + \frac{v_B^2}{2g} = Z_C + \frac{p_C}{\gamma} + \frac{v_C^2}{2g}$$

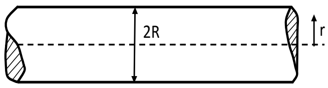
$$0 + 0.23 + \frac{v_B^2}{2g} = 0 + 10 + \frac{(4.42)^2}{19.62} \rightarrow v_B^2 = \rightarrow v_B = \sqrt{211.30} \rightarrow v_B = 14.54\, m/s$$

$$Q = v_B x A_B \rightarrow 0.14 = 14.54x\frac{(d_{min})^2\pi}{4} \rightarrow d_{min} = 0.14\, m$$



SOLUTIONS

Question 2: The velocity distribution on the cross-section of a pipe of 10 cm diameter is given in metric units as $U = 400\left(R^2 - r^2\right)$. Find the maximum velocity on the axis, discharge of the pipe and average velocity in the pipe.



Solution 2:

R=10 cm

$$U = 400(R^2 - r^2)$$

$$U_{max} = ?\,;\, Q = ?\,;\; V = ?$$

$$For\ U_{max}\ r\ should\ be\ zero\ r = 0$$

$$U_{max} = 400((0.1)^2 - 0^2) = 4\,m/s$$

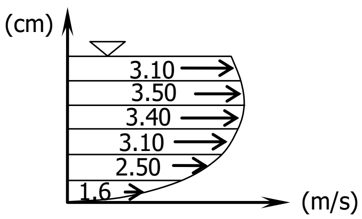
$$Q = \int_A udA\,;\, A = \pi r^2\,;\, dA = 2\pi r dr$$

$$Q = \int_0^R u2\pi r dr \ \rightarrow Q = 2\pi \int_0^{0.1} 400((0.1)^2 - 0^2) r dr$$

$$Q = 2\pi \int_0^{0.1} 400 r (0.1)^2 dr - 2\pi \int_0^{0.1} 400 r^3 dr \ \rightarrow Q = 0.0628\,m^3/s$$

$$V = \frac{0.0628}{\pi(0.1)^2} \rightarrow V = 2\,m/s$$

Question 3: Horizontal velocity measurements made by a pitot tube along a vertical line in the mid-sections of a wide channel is shown below. Calculate the channel's discharge per unit width and its average discharge.



Solution 3:

$$q_1 = v_1 A_1 \ \rightarrow\ q_1 = 1.60x(1x0.5) \ \rightarrow\ q_1 = \frac{1.6}{2} m^3/s\,.\,m$$

$$q_2 = v_2 A_2 \ \rightarrow\ q_2 = 2.50x(1x0.5) \ \rightarrow\ q_2 = \frac{2.50}{2} m^3/s\,.\,m$$

$$q = \frac{1}{2}(1.60 + 2.50 + 3.10 + 3.40 + 3.50 + 3.10) \ \rightarrow q = 8.6\,m^3/s\,.\,m$$

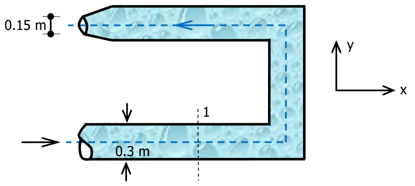
$$V_{avg} = \frac{q_t}{A_t} \ \rightarrow\ V_{ort} = \frac{8.6}{1x3} \rightarrow V_{ort} = 2.87\,m/s$$

Question 4: A water jet flowing through a horizontal elbow shown in the figure given below is poured into the atmosphere. Average flow velocity at cross-section (1) is v_1=2 m/s and gage pressure is p_1=19.62 N/cm^2. With the assumptions of ideal and incompressible fluid and taking absolute atmospheric as 9.81 N/cm^2:

a- Find the energy loss at the elbow.

SOLUTIONS

b- Find the x,y components of the force that the flow exerts on the elbow.



Solution 4:

$v_1 = 2\, m/s\, ; p_1 = 19.62\, N/cm^2\, ;\ p_0 = 9.81\, N/cm^2$

a-

$Q = 2\frac{\pi(0.3)^2}{4} \rightarrow Q = 0.1414 m^3/s$

$v_2 = \frac{0.1414}{\pi(0.15)^2} \rightarrow\ v_2 = 8\, m/s$

$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_k$

$\frac{2^2}{19.62} + 20 = \frac{8^2}{19.62} + h_k\ \rightarrow\ h_k = 16.94\, m$

b-

$p_1 A_1 = 196.2\frac{\pi\,(0.3)^2}{4} = 13.87\, kN$

$\rho Q v_1 = \frac{9.81x1000}{9.81} x0.1414x2 = 0.28\, kN$

$\rho Q v_2 = \frac{9.81x1000}{9.81} x0.1414x8 = 1.13\, kN$

$\sum F_{xi} = p_1 A_1 + \rho Q v_1 + \rho Q v_2 - R_x = 0$

$R_x = 13.87 + 0.28 + 1.13\ \rightarrow\ R_x = 15.28\, kN$

$R_x = \ -15.28\, kN$

Question 5: Velocity components of an ideal and incompressible fluid in a two-dimensional flow (2D) is given as

$u = -2\,a\,x\,,\quad v = -2\,a\,y$ (a=constant).

a- Is such a flow physically possible?

b- Is there a velocity potential for this function? If so, find the velocity potential function.

c- Find the stream function for this flow.

d- For a=1, find the resultant velocity and acceleration and their components at point M(1,1).

Solution 5:

$u = -2ax;\ \ v = 2ay$

a-

$It\ has\ to\ be\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$\frac{\partial u}{\partial x} = -2a$

SOLUTIONS

$$\frac{\partial v}{\partial y} = 2a$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow -2a + 2a = 0 \ \rightarrow Flow\ is\ physically\ possible.$$

b-

$$w_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial x}\right) \ ; \ w_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right); \ w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \ \rightarrow \ \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

$$\frac{\partial v}{\partial x} = 0; \ \frac{\partial u}{\partial y} = 0 \ \rightarrow 0 = 0 \ \ There\ exits\ velocity\ potential. Therefore, the\ flow\ is\ irrotational.\ Hence,$$

$$u = \frac{\partial \emptyset}{\partial x}, v = \frac{\partial \emptyset}{\partial y}$$

$$\partial\emptyset_1 = u\partial x \ \rightarrow \int \partial\emptyset_1 = \int -2ax\partial x \ \rightarrow \ \emptyset_1 = -ax^2 + c_1$$

$$\partial\emptyset_2 = v\partial y \ \rightarrow \int \partial\emptyset_2 = \int -2ay\partial y \ \rightarrow \ \emptyset_2 = ay^2 + c_2$$

$$\emptyset = \emptyset_1 + \emptyset_2 \ \rightarrow \ \emptyset = a(y^2 - x^2) + c$$

c-

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$

$$\int \partial\psi_1 = \int u\, \partial y \rightarrow \int \partial\psi_1 = \int -2ax\, \partial y \ \ \rightarrow \psi_1 = -2axy + c_1$$

$$\int \partial\psi_2 = \int -v\, \partial x \rightarrow \int \partial\psi_2 = \int -2ay\, \partial x \ \ \rightarrow \psi_2 = -2axy + c_2$$

$$\psi = \psi_1 + \psi_2 \rightarrow \psi = -2axy + c$$

d-

For a=1, M(1,1)

$$u = -2\, m/s\, ; v = 2\, m/s \ \rightarrow V = \sqrt{(-2)^2 + (2)^2}$$

$$V = 2\sqrt{2}\, m/s$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \ \rightarrow \ a_x = -2(-2a) + 2(0) + 0 \ \rightarrow \ a_x = 4\, m/s^2$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \ \rightarrow \ a_y = -2(0) + 2(2y) + 0 \ \rightarrow \ a_y = 4\, m/s^2$$

$$a = \sqrt{(a_x)^2 + \left(a_y\right)^2} \ \rightarrow a = \sqrt{(4)^2 + (4)^2} \ \rightarrow a = 4\sqrt{2}\, m/s^2$$

Question 6: The velocity components for a two-dimensional (2D) incompressible flow on the (x-y) plane is given as $u = -x, \quad v = y$.

- **a-** Find the stream function for this flow.
- **b-** Is there a velocity potential for this function? If so, find the velocity potential function.

SOLUTIONS

c- For this flow, find the discharge per unit width that passes from a line or a curvature which connects the points A(-1,1) and B(-2,3).

Solution 6:

$u = -x, v = y$

a-

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$

$$\partial \psi = -x\partial y \rightarrow \int \partial \psi = \int -x\partial y \rightarrow \psi_1 = -xy + c_1$$

$$\partial \psi = -y\partial y \rightarrow \int \partial \psi = \int -y\partial y \rightarrow \psi_2 = -xy + c_2$$

$$\psi = \psi_1 + \psi_2 = -xy + c \rightarrow \psi = -xy + c = sabit$$

b-

We have to satisfy irrotationality condition. $\emptyset = \emptyset(x, y, t)$

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \; irrotationality\ condition$$

$$\frac{\partial v}{\partial x} = 0; \frac{\partial u}{\partial u} = 0 \rightarrow \frac{\partial v}{\partial x} - \frac{\partial u}{\partial u} = 0 \rightarrow 0 - 0 = 0. There\ exits\ velocity\ potential\ u = \frac{\partial \emptyset}{\partial x}, v = \frac{\partial \emptyset}{\partial y}$$

$$\int \partial \emptyset = \int -x\partial x \rightarrow \emptyset_1 = -\frac{x^2}{2} + c_1$$

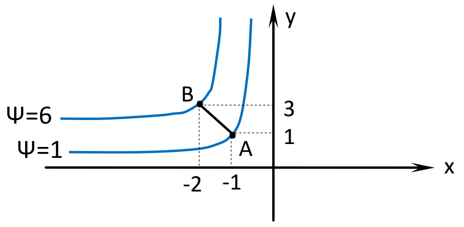
$$\int \partial \emptyset = \int y\partial y \rightarrow \emptyset_2 = -\frac{y^2}{2} + c_2$$

$$\emptyset = \emptyset_1 + \emptyset_2 \rightarrow \emptyset = \frac{1}{2}(y^2 - x^2) + c$$

c-

$$A(-1,1) \rightarrow \psi_A = -1(-1)(1) = 1$$

$$B(-2,3) \rightarrow \psi_B = -1(-2)(3) = 6$$



$$q = \int_A^B d\psi \rightarrow q = \psi_A - \psi_B \rightarrow q = (6-1)1 \rightarrow q = 5m^3/s\,.m$$

Question 7: The stream function of a two-dimensional (2D) ideal and incompressible flow is given as $\Psi = -2\,a\,x\,y$.

a- Is such a flow physically possible?

b- Is there a velocity potential for this function ? If so, find the velocity potential function.

c- For a=1, find the resultant velocity and acceleration and their components at point N(1,1).

d- Draw the flow net.

SOLUTIONS

Solution 7:

a-

$$u = \frac{\partial \psi}{\partial y} = -2ax$$

$$v = -\frac{\partial \psi}{\partial x} = 2ay$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = o \; Is \; it \; correct?$$

$$-2a + 2a = 0 \; satisfies \; the \; continuity \; equation \; and \; is \; physically \; possible.$$

b-

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \;\rightarrow\; w_z = \frac{1}{2}(0-0) = 0 \; irrotational \; (existing \; velocity \; potential)$$

$$u = \frac{\partial \emptyset}{\partial x} \;\rightarrow\; \partial\emptyset = -2ax\partial x \;\rightarrow \int \partial\emptyset = \int -2ax\partial x \;\rightarrow\; \emptyset_1 = -ax^2 + c_1$$

$$v = \frac{\partial \emptyset}{\partial y} \;\rightarrow\; \partial\emptyset = 2ay\partial y \;\rightarrow \int \partial\emptyset = \int 2ay\partial y \;\rightarrow \emptyset_2 = ay^2 + c_2$$

$$\emptyset = \emptyset_1 + \emptyset_2 \;\rightarrow\; \emptyset = a(y^2 - x^2) + c$$

c-

$$a = 1 = constant \; and \; N(1,1) \;\rightarrow u = -2 \; ve \; v = 2 \;\rightarrow V = \sqrt{(u^2) + (v^2)} \;\rightarrow V = \sqrt{(-2^2) + (2^2)} \;\rightarrow V = 2\sqrt{2} \; m/s$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} \;\rightarrow\; a_x = (-2)(-2) + (2)(0) \;\rightarrow\; a_x = 4 \; m/s^2$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \;\rightarrow\; a_y = (-2)(0) + (2)(2) \;\rightarrow\; a_y = 4 \; m/s^2$$

$$a = \sqrt{(a_x)^2 + (a_y)^2} \;\rightarrow a = \sqrt{(4)^2 + (4)^2} \;\rightarrow a = 4\sqrt{2} \; m/s$$

d-

$$Let \; \psi = -4a$$

$$-2axy = -4a$$

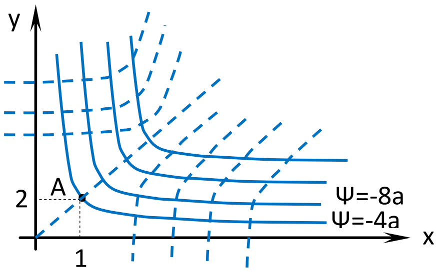
$$xy = 2 \;\rightarrow y = \frac{2}{x}$$

$$-2axy = -8a$$

$$xy = 4$$

$$y = \frac{4}{x}$$

SOLUTIONS



Question 8: A two-dimensional (2D) flow is given with components $u = 4y$, $v = 4x$.

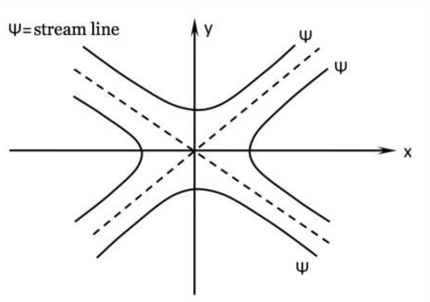
a- Draw the streamlines of this flow.

b- Calculate the acceleration components at point x=1, y=1.

c- Find the stream function and the potential function of this flow (if there is one).

Solution 8:

a-



a-

Acceleration components at point x=1 ve y=1

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial u}{\partial t} \rightarrow \frac{\partial u}{\partial x} = 0; \frac{\partial u}{\partial y} = 4; \frac{\partial v}{\partial x} = 4; \frac{\partial v}{\partial y} = 0; \frac{\partial u}{\partial t} = 0; \frac{\partial v}{\partial t} = 0$$

$$a_x = (4)(0) + (4)(4) + 0 \rightarrow a_x = 16m/s^2$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}$$

$$a_y = (4)(4) + (4)(0) + 0 \rightarrow a_y = 16\ m/s^2$$

$$Resultant\ acceleration\ a = \sqrt{(a_x)^2 + (a_y)^2} \rightarrow a = \sqrt{(16)^2 + (16)^2} \rightarrow a = 16\sqrt{2}\ m/s^2$$

b-

$$u = \frac{\partial \psi}{\partial y} \rightarrow \int \partial\psi = \int u\partial y \rightarrow \int \partial\psi = \int 4y\partial y \rightarrow \psi_1 = 2y^2 + c_1$$

$$v = -\frac{\partial \psi}{\partial x} \rightarrow \int \partial\psi = \int -v\partial x \rightarrow \int \partial\psi = \int -4x\partial x \rightarrow \psi_2 = -2x^2 + c_2$$

$$\psi = \psi_1 + \psi_2 \rightarrow \psi = 2y^2 + c_1 + (-2x^2) + c_2$$

$$\psi = 2(y^2 - x^2) + c \quad stream\ function$$

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \rightarrow w_z = \frac{1}{2}(4 - 4) = 0\ \ existing\ velocity\ potential.$$

SOLUTIONS

$$u = \frac{\partial\emptyset}{\partial x} \rightarrow \int \partial\emptyset = \int u\partial x \rightarrow \int \partial\emptyset = \int x\partial x \rightarrow \emptyset_1 = 4xy + c_1$$

$$v = \frac{\partial\emptyset}{\partial y} \rightarrow \int \partial\emptyset = \int v\partial y \rightarrow \int \partial\emptyset = \int 4x\partial y \rightarrow \emptyset_2 = 4xy + c_2$$

$$\emptyset = \emptyset_1 + \emptyset_2 \rightarrow \emptyset = 4xy + c_1 + 4xy + c_2$$

$$\emptyset = 4xy + c \;\; potential\; function.$$

Question 9: Velocity components of an incompressible liquid are as follows.

$$u = k\,x(y+z) \quad , \quad v = k\,y(x+z) \quad , \quad w = -k\,z(x+y) - z^2$$

- a- What should be the "k" for the given velocity field to correspond to a possible velocity field of a fluid?
- b- Is the flow steady (permanent)? Why?
- c- Is the flow uniform? Why?
- d- Is the flow rotational? Why?
- e- Calculate the components of the rotation vector at point (1, -1, 1).

Solution 9:

a- What should be the "k" for the given velocity field to correspond to a possible velocity field of a fluid? It has to satisfy continuity equation.

$$which\; is \;\; \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$k(y+z) + k(x+z) - k(x+y) = 0$$

$$k(y + z + x + z - z - y) - 2z = 0$$

$$2zk - 2z = 0 \;\rightarrow 2zk = 2z$$

$$k = 1\,.$$

b-

If the flow is independent of time, then STEADY-STATE (PERMENANT) flow exists.

$$\frac{\partial u}{\partial t} = 0; \frac{\partial v}{\partial t} = 0; \frac{\partial w}{\partial t} = 0 \;\; When\; the\; velocity\; components\; of\; the\; flow\; are\; independent\; of\; t\; (time)$$

$$the\; flow\; is\; steady - state\; (permant).$$

c-

The flow is uniform if its characteristics are the same along the flow $\frac{\partial v}{\partial x} = 0 \; if \; \frac{\partial p}{\partial x} = 0$.

$$\frac{\partial u}{\partial x} = k(y+z); varying$$

$$\frac{\partial v}{\partial y} = k(x+z); varying$$

$$\frac{\partial w}{\partial z} = -k(x+y) - 2z; varying$$

The flow is not uniform.

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

$$If\; w_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = 0 \;\; it\; is\; irrotational\; flow$$

SOLUTIONS

$$w_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = 0$$

d-

$$w_z = \frac{1}{2}(y - x) \neq 0$$

$$w_y = \frac{1}{2}(x + z) \neq 0 \ \ rotational\ flow.$$

$$w_z = \frac{1}{2}(-z - y) \neq 0$$

e-

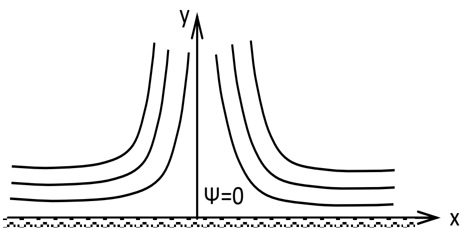
$$w_z = \frac{1}{2}(-1 - 1) = -1$$

$$w_y = \frac{1}{2}(1 + 1) = 1$$

$$w_z = \frac{1}{2}(-1 + 1) = 0$$

Question 10: If the vertical velocity component of a two-dimensional water jet hitting a horizontal plate is proportional to the distance to the plate, find the stream function that defines the flow field.

Solution 10:



If we observe the graph: it could be written as $v = -ky$ or $\frac{\partial v}{\partial y} = -k.$

Moreover, according to the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\ ,$$

$$\partial u = -\frac{\partial v}{\partial y}dx \ \rightarrow \int \partial u = -\int (-k)dx \rightarrow u = kx + c$$

As a boundary condition, because of symmetry, if we take u=0 for x=0, we will end up c=0.
Since the exact differential of stream line is

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy \ \rightarrow d\psi = -vdx + udy\ if\ we\ plu\ in\ the\ values\ of\ u\ and\ v\ we\ will\ have:$$

$$d\psi = kydx + kxdy \ \rightarrow \ d\psi = k(ydx + xdy)\ integrating\ both\ sides,$$

$$it\ becomes \int d\psi = \int kydx + \int xdy \ \rightarrow \ \psi = kxy + c\ .$$

The appearance of the streamlines:
Since ψ=constant along a streamline, from the last expression we get,

SOLUTIONS

$y = \frac{constant}{x}$ According to this expression, the streamlines are hyperbolic. Besides, for the streamlines along x and y axis, it becomes x=0, y=0.

Question 11: The velocity field of an incompressible fluid in a planar flow is:

$u = 3x^2 - 3y^2$ and $v = -6\,x\,y$

a- Show whether the flow is irrotational.

b- Write the resultant acceleration and their components at point M(x,y). Find the resultant acceleration at point A(1,1).

Solution 11:

a- It has to be:

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$$

$$w_z = \frac{1}{2}\big(-6y - (-6y)\big) = 0 \ \ The\ flow\ is\ irrotational.$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} \ \rightarrow\ a_x = (3x^2 - 3y^2)(6x) + (-6xy)(-6y) \ \rightarrow\ a_x = 18x(x^2 + y^2)$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial v}{\partial t} \ \rightarrow\ a_y = (3x^2 - 3y^2)(6y) + (-6xy)(-6x) \ \rightarrow\ a_y = 18y(x^2 + y^2)$$

b-

$$a_x = 18(1)((1)^2 + (1)^2) \ \rightarrow\ a_x = 36\,m/s^2$$

$$a_y = 18(1)((1)^2 + (1)^2) \ \rightarrow\ a_y = 36\,m/s^2$$

$$a = \sqrt{(36)^2 + (36)^2} \ \rightarrow a = 36\sqrt{2}\,m/s^2$$

Question 12: The velocity field of a two-dimensional (2D) flow is given as:

$u = \left(2\,x\,y + t^2\right)$, $v = \left(x^2 - y^2 + 10t\right)$

a- Is such a flow physically possible?

b- Is the flow steady (permanent)?

c- Is there a velocity potential for this function? If so, find out the velocity potential function.

d- Find the stream function of this flow.

e- In this flow field, find the resultant velocity and acceleration and their components at point A(1,1) at time t=1.

Solution 12:

a- It has to be :

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$2y - 2y = 0 \ \ physically\ possible.$$

b-

Since there are terms dependent of t (time) in the equations of the velocity components of the flow, the flow is not steady-state (not permanent).

$$\frac{\partial u}{\partial t} = 2t \neq 0\,, \frac{\partial u}{\partial t} = 10 \neq 0 \ \ so\ it\ is\ not\ permenant.$$

c-

SOLUTIONS

$$w_z=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\rightarrow\ w_z=(2x-2y)=0\ \ existing\ velocity\ potential.$$

$$u=\frac{\partial\emptyset}{\partial x}\ \rightarrow\ \int\partial\emptyset=\int u\partial x\ \rightarrow\ \int\partial\emptyset=\int(2xy+t^2)\partial x\ \rightarrow\ \emptyset_1=x^2y+t^2y+c_1$$

$$v=\frac{\partial\emptyset}{\partial y}\ \rightarrow\ \int\partial\emptyset=\int v\partial y\ \rightarrow\ \int\partial\emptyset=\int(x^2+y^2+10t)\partial y\ \rightarrow\ \emptyset_2=x^2y-\frac{1}{3}y^3+10ty+c_2$$

$$\emptyset=\emptyset_1+\emptyset_2\ \rightarrow\ \emptyset=x^2y-\frac{1}{3}y^3+t(tx+10y)+c$$

d-

$$u=\frac{\partial\psi}{\partial y}\rightarrow\ \int\partial\psi=\int u\partial y\ \rightarrow\ \int\partial\psi=\int(2xy+t^2)\partial y\ \rightarrow\psi_1=xy^2+t^2y+c_1$$

$$v=-\frac{\partial\psi}{\partial x}\rightarrow\ \int\partial\psi=\int-v\partial x\ \rightarrow\ \int\partial\psi=\int-(x^2-y^2+10t)\partial x\ \rightarrow\psi_2=-\frac{x^3}{3}+xy^2-10tx+c_2$$

$$\psi=\psi_1+\psi_2\ \rightarrow\ \psi=xy^2+t^2y+c_1-\frac{x^3}{3}+xy^2-10tx+c_2$$

$$\psi=-\frac{x^3}{3}+xy^2+t(ty-10x)+c$$

e-

$$\frac{\partial u}{\partial t}=2t\ \rightarrow t=1\ \rightarrow\ \frac{\partial u}{\partial t}=2\ local\ acceleration$$

$$\frac{\partial v}{\partial t}=10\ \rightarrow local\ acceleration$$

$$\frac{\partial u}{\partial x}=2x\ \rightarrow x=1\ \rightarrow\ \frac{\partial u}{\partial x}=2$$

$$\frac{\partial u}{\partial y}=2y\ \rightarrow y=1\ \rightarrow\ \frac{\partial u}{\partial y}=2\quad convective\ acceleration$$

$$\frac{\partial v}{\partial x}=2x\ \rightarrow x=1\ \rightarrow\ \frac{\partial v}{\partial x}=2$$

$$\frac{\partial v}{\partial y}=-2y\ \rightarrow y=1\ \rightarrow\ \frac{\partial v}{\partial y}=-2$$

$$a_x=\frac{du}{dt}=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\ \rightarrow\ a_x=2+(3)(2)+(10)(2)\ \rightarrow\ a_x=28\,m/s^2$$

$$a_y=\frac{dv}{dt}=\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\ \rightarrow\ a_y=10+(3)(2)+(10)(-2)\ \rightarrow\ a_y=-4\,m/s^2$$

$$a=\sqrt{(28)^2+(-4)^2}\ \rightarrow a=28.28\,m/s^2$$

Question 13: The velocity components of an ideal fluid in a two-dimensional (2D) flow are given as:

$$u=16y-12x\quad,\qquad v=12y-9x$$

For this flow:

a- Show whether the flow steady (permanent) or not.

b- Determine whether such a flow is physically possible or not.

c- Examine whether a velocity potential exists or not?

d- Find the stream function and find the equation of a streamline that passes through the point which has coordinates x=1, y=2.

SOLUTIONS

e- Is it possible to determine the equation of equi-potential lines? Explain why.

f- Explain where the Bernoulli equation is valid for this flow.

Solution 13:

a-

If the flow is permanent, it has to satisfy $\frac{\partial u}{\partial t} = 0, \frac{\partial v}{\partial t} = 0$

Since the u and v are independent of t (time) $\frac{\partial u}{\partial t} = 0 \; and \; \frac{\partial v}{\partial t} = 0.$ Therefore, the flow is permanent.

b-

For such a flow to exist Physically, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \mathbf{0}$.

$$\frac{\partial u}{\partial x} = -12$$

$$\frac{\partial v}{\partial y} = \mathbf{12}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \;\rightarrow\; \mathbf{-12 + 12 = 0}$$

Therefore, it is possible to have this flow physically.

c-

$$w_z = \tfrac{1}{2}\left(\tfrac{\partial v}{\partial x} - \tfrac{\partial u}{\partial y}\right) = 0 \;\rightarrow Existing\ velocity\ potential.$$

$$\frac{\partial v}{\partial x} = -9$$

$$\frac{\partial u}{\partial y} = 16$$

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \rightarrow \; w_z = \frac{1}{2}(-9 - 16) \neq 0 \;\; No\ velocity\ potential\ exists.$$

d-

Function of the streamline:

$$u = \frac{\partial \psi}{\partial y} \rightarrow \int \partial\psi = \int u\partial y \;\rightarrow \int \partial\psi = \int (16y - 12x)\partial y \;\rightarrow \psi_1 = 8y^2 - 12xy + c_1$$

$$v = -\frac{\partial \psi}{\partial x} \rightarrow \int \partial\psi = \int -v\partial x \;\rightarrow \int \partial\psi = \int -(12y - 9x)\partial x \;\rightarrow \psi_2 = 12xy - \frac{9x^2}{2} + c_2$$

$$\psi = \psi_1 + \psi_2 \;\;\rightarrow\; \psi = 8y^2 - 12xy + c_1 + 12xy - \frac{9x^2}{2} + c_2$$

$$\psi = 8y^2 - \frac{9}{2}x^2 + c$$

e- It cannot be determined. Because velocity potential does not exist.

f- Since the flow is irrotational, the BERNOULLI equation is only valid along a streamline.