Kinematics

Question 1: A flow field is given as $\vec{V} = -x.\vec{\imath} + 2y.\vec{\jmath} + (5-z).\vec{k}$. Find the equation of the streamlines on the projections of x-y, y-z, x-z planes and calculate the x, y and z components of the acceleration field.

Answer 1 :

$\vec{V} = -x.\vec{\imath} + 2y.\vec{\jmath} + (5-z).\vec{k}$

Streamline equations: $\frac{dx}{u(x,y,z,t_1)} = \frac{dy}{v(x,y,z,t_1)} = \frac{dz}{w(x,y,z,t_1)}$

<u>Equation on the projection of x-y plane:</u>

$$\int \frac{dx}{-x} = \int \frac{dy}{2y} \Rightarrow -2lnx = lny + lnc \Rightarrow -2lnx - lny = lnc \Rightarrow ln\frac{1}{x^2y} = lnc \Rightarrow c = \frac{1}{x^2y}$$

<u>Equation on the projection of y-z plane:</u>

$$\int \frac{dy}{-x} = \int \frac{dz}{5-z} \Rightarrow lny(5-z)^2 = lnc \Rightarrow y = \frac{c}{(5-z)^2}$$

<u>Equation on the projection of x-z plane:</u>

$$\int \frac{dx}{-x} = \int \frac{dz}{(5-z)} \Rightarrow ln\left[\frac{1}{x}.(5-z)\right] = lnc \Rightarrow c = \frac{5-z}{x}$$

<u>x component of acceleration</u>:

$$a_x = \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} = -x.(-1) + 2y.0 + (5-z).0 = x = -u$$

<u>y component of acceleration</u>:

$$a_y = \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} = -x.0 + 2y.2 + (5-z).0 + 0 = 4y = 2v$$

<u>z component of acceleration</u>:

$$a_z = \frac{dw}{dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} = -x.(-1) + 2y.0 + (5-z).0 = -(5-z) = -w$$

Kinematics

Question 2: A velocity field is given as velocity components $u=V.\cos\theta$, $v=V.\sin\theta$ and $w=0$. Find the equation of the streamline by taking V and θ as constants.

Answer 2:

Stereamline equation: $\frac{dx}{u(x,y,z,t_1)} = \frac{dy}{v(x,y,z,t_1)} = \frac{dz}{w(x,y,z,t_1)}$

$$\frac{dx}{u} = \frac{dy}{v} \Rightarrow \frac{dx}{V.\cos\theta} = \frac{dy}{V.\sin\theta}$$

$$\Rightarrow \int \frac{\sin\theta}{\cos\theta} dx = \int dy \Rightarrow \tan\theta . x + c_1 = y + c_2 \Rightarrow y = x.\tan\theta + c$$

Question 3: Velocity components of a 2-dimensional (2-D) steady (permanant) flow field is given as $u=x^2-y^2$, $v=-2xy$. Find the equation of streamline.

Answer 3:

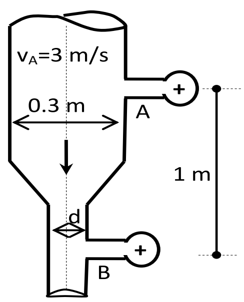
Stereamline equation: $\frac{dx}{u(x,y,z,t_1)} = \frac{dy}{v(x,y,z,t_1)} = \frac{dz}{w(x,y,z,t_1)}$

$$\frac{dx}{u} = \frac{dy}{v} \Rightarrow \frac{dx}{x^2 - y^2} = \frac{dy}{-2xy} \Rightarrow -2xydx = (x^2 - y^2)dy$$

$$\int df = \int 2xydx + \int (x^2 - y^2)dy \Rightarrow f(x,y) = \frac{2x^2y}{2} - \frac{y^3}{3} + x^2y + c \Rightarrow f(x,y)$$

$$= 2x^2y - \frac{y^3}{3} + c \Rightarrow 2x^2y - \frac{y^3}{3} = c$$

Question 4: Find the diameter (d) of pipe B by neglecting energy losses and if the manometers given in the figures show the same pressure value. (It should be considered that the pipe is horizontal).



Answer 4:

$v_A = 3\,m/s\,, d_A = 0.3m, d_B = ?$

From contuniuty equation:

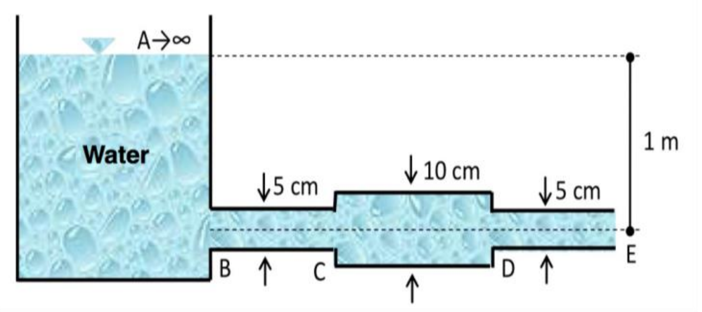
$$Q = v_A . A_A = v_B A_B \Rightarrow 3x\frac{\pi(0.3)^2}{4} = v_B \frac{\pi {d_B}^2}{4} \Rightarrow v_B = \frac{0.27}{{d_B}^2}$$

If we write Bernoulli equation between A-B:

$$Z_A + \frac{P_A}{\gamma} + \frac{{v_A}^2}{2g} = Z_B + \frac{P_B}{\gamma} + \frac{{v_B}^2}{2g}$$

$$\frac{{v_A}^2}{2g} + (z_A - z_B) = \frac{{v_B}^2}{2g} \rightarrow \frac{(3)^2}{19.62} + 1 = \frac{{v_B}^2}{19.62} \rightarrow v_B = 5.35\,m/s \rightarrow d_B = 0.225m$$

Question 5: Calculate the discharge of the chamber-pipe system by considering the liquid as ideal (inviscid). Draw the gage energy and gage piezometer lines.



Answer 5:

If we write Bernoulli equation between A-E:

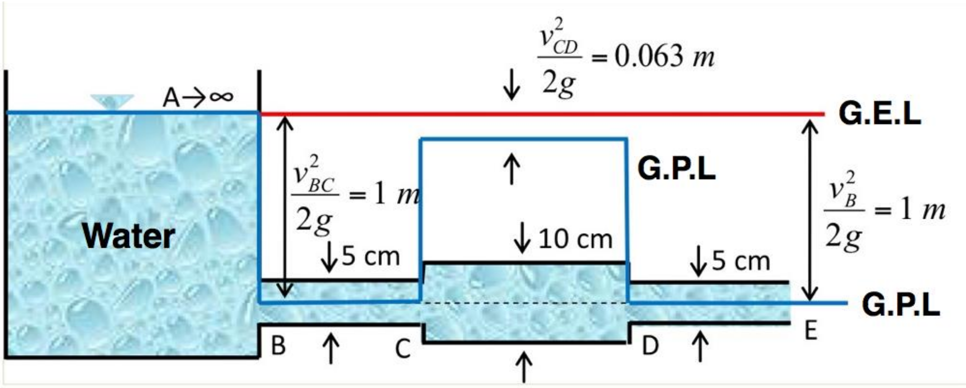
$$Z_A + \frac{P_A}{\gamma} + \frac{{v_A}^2}{2g} = Z_E + \frac{P_E}{\gamma} + \frac{{v_E}^2}{2g}$$

$$1 + 0 + 0 = 0 + 0 + \frac{{v_E}^2}{2g} \rightarrow 1 = \frac{{v_E}^2}{19.62} \rightarrow v_E = 4.43\,m/s$$

$$D_{BC} = D_{DE} \rightarrow v_{BC} = v_{DE} = 4.43\,m/s \rightarrow \frac{{v_{BC}}^2}{2g} = \frac{{v_{DE}}^2}{2g} = 1m$$

$$Q = v_{DE} A_{DE} \rightarrow Q = 4.43\frac{3.14(0.05)^2}{4} \rightarrow Q = 0.0087m^3/s$$

$$0.0087 = {v_{CD}}^2 \frac{\pi(0.1)^2}{4} \rightarrow v_{CD} = 1.11m/s$$



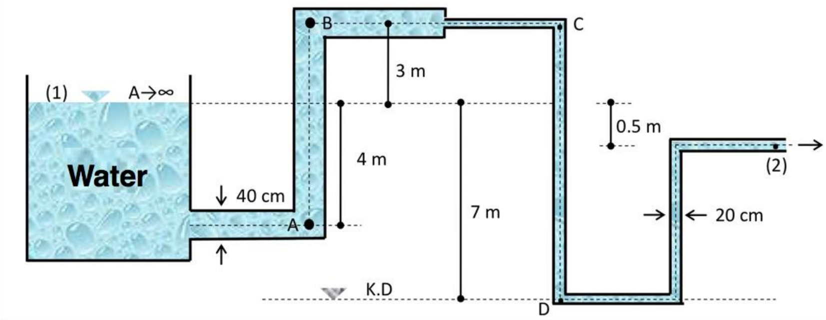
Question 6: For the reservoir–pipe system shown in the figure given below, find:

a- The discharges in the pipes,

b- Velocities and the pressures at points A, B, C,and D.

c- Draw the gage and absolute energy and piezometer lines of the system.

d- Since the absolute vapor pressure of water is 2.26 kN/m^2, what should be the maximum value of h?



Answer 6:

a) If we write Bernoulli equation between A-E:

$$z_1 + \frac{P_1}{\gamma} + \frac{{v_1}^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{{v_2}^2}{2g} \rightarrow 7 = \frac{{v_2}^2}{2g} + 6.5 \rightarrow v_2 = 3.13 m/s$$

$$Q = v_2 A_2 \rightarrow Q = 3.13x\frac{\pi(0.2)^2}{4} = 0.0984 m^3/s$$

$Q = v_A A_A = v_2 A_2$ Continuity Equation

$$0.0984 = v_A \frac{\pi(0.4)^2}{4} \rightarrow v_A = 0.783\, m/s \ \rightarrow D_A = D_B \rightarrow v_A = v_B$$

$$D_2 = D_D = D_C \rightarrow v_2 = v_D = v_C = 3.13\ m/s$$

$$\frac{v_A{}^2}{2g} = 0.0313m, \frac{v_2{}^2}{2g} = 0.5m$$

To find the pressures, we will use Bernoulli Equation between 1-A

$$z_1 + \frac{P_1}{\gamma} + \frac{v_1{}^2}{2g} = z_A + \frac{P_A}{\gamma} + \frac{v_A{}^2}{2g} \rightarrow 7 + 0 + 0 = \frac{(0.783)^2}{2g} + \frac{P_A}{\gamma} + 3 \rightarrow \frac{P_A}{\gamma} = 3.97m \rightarrow P_A = 38.95kN/m^2$$

b)

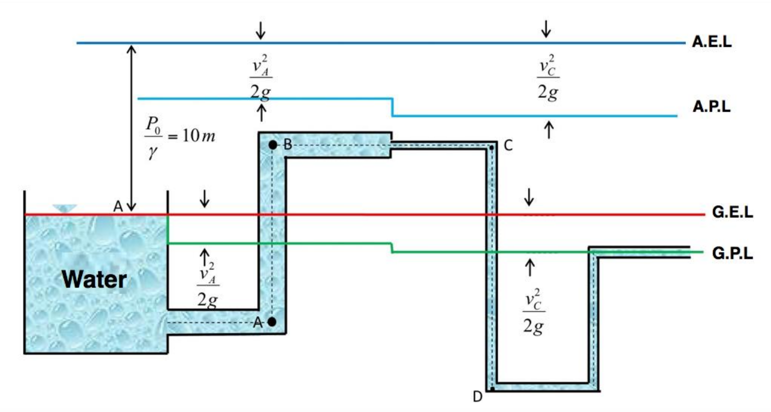
In the same manner:

Writing Bernoulli between 1-B $\rightarrow P_B = -29.72\ kN/m^2$

Writing Bernoulli between 1-C $\rightarrow P_C = -34.34\ kN/m^2$

Writing Bernoulli between 1-D $\rightarrow P_D = 63.77\ kN/m^2$

c)



d) If we write Bernoulli equation between 1-C ;

$$z_1 + \frac{P_1}{\gamma} + \frac{v_1{}^2}{2g} = z_C + \frac{P_C}{\gamma} + \frac{v_C{}^2}{2g} \rightarrow 10 + 0 + 0 = 0.5 + 0.23 + z_C \rightarrow z_C = 9.27m$$