Relative Equilibrium

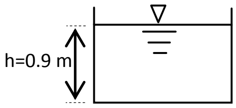
Question 1: A tank which has a liquid with a specific weight $\gamma = 9.22$ kN/m^3 inside it has an upward constant acceleration of 4.8 m/s^2. Depth of the liquid in the tank is 0.9 meters. Dimensions of the base of the tank are 1.20 x 1.50 meters. Find the pressure and the pressure force at the base of the tank

a. when the tank is accelerating,

b. after the tank's acceleration dies out and when it keeps moving upward with a constant velocity of 6 m/s.

Solution 1:

a) When the tank is accelerating:



$$m = \rho.V = \frac{\gamma}{g}.V = \frac{9.22}{9.81}x0.9x1.20x1.50 = \frac{1.57kNs^2}{m}$$

$$a_T = a + g = 4.8 + 9.81 = 14.61m/s^2$$

$$p = \gamma.h \;\rightarrow\; \rho.a_T.h \;\rightarrow p = \frac{9.22}{9.81}x14.61x0.9 \;\rightarrow p = 12.36\ kN/m^2$$

$$S = 1.2mx1.5m = 1.8m^2$$

$$F = p.S = 12.36x1.8 = 22.27\ kN$$

b) When the tank is moving with a constant velocity:

Constant velocity ⇒ acceleration is zero; pressure distribution is hydrostatic. Therefore,

$$p = \gamma.h = 9.22x0.9 = 8.34\ kN/m^2$$

$$F = p.S = 8.34x1.2x1.5 = 14.91\ kN$$

Question 2: A container that is partially filled with water is dragged with an acceleration of a=4 m/s^2 at an angle of 30^o with horizontal plane. Given that the container's base width is 4 meter and the depth of the water before motion has started is 1.5 meter,

a. Calculate the angle of the water's surface with horizontal plane.

b. Calculate the maximum and minimum pressures on the base (bottom) of the container.

Solution 2:

a) Angle of the water's surface with horizontal plane:

$$tan\theta = \frac{a_x}{a_y + g} = \frac{a.cos\alpha}{a.sin\alpha + g} = \frac{4.cos30°}{4.sin30° + 9.81} = \frac{3.46}{11.81} = 0.29$$

$\theta = 16.33°$

b) Maximum and minimum pressures on the base (bottom) of the container:

$$h_{max} = 1.5 + 2.tan(16.33°) = 2.09m$$

$$h_{min} = 1.5 - 2.tan(16.33°) = 0.91m$$

$$p_{max} = \gamma.h_{max}\left(1 + \frac{a_y}{g}\right) = 9.81x2.09x(1 + 2/9.81) = 24.72kN/m^2$$

$$p_{min} = \gamma.h_{min}\left(1 + \frac{a_y}{g}\right) = 9.81x0.91x(1 + 2/9.81) = 10.79kN/m^2$$

Question 3: The depth of water in an open–topped cylindrical container is 1.5 meter. The container is being rotated with angular velocity ω around its own axis.

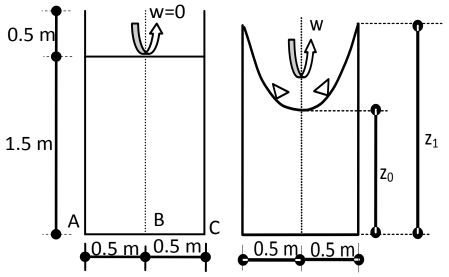
a. Calculate the maximum angular velocity of the container that could be attained without spilling the water.

b. Calculate the maximum angular velocity that could be attained while keeping the water depth above the container's axis to be $Z_0 = 0$.

c. Find the pressure values on the bottom and on the sides B and C for ω= 6 rad/s.

Note: Volume of the paraboloid is half of the cylinder's volume that is built right on it.

Solution 3:



a) Water not to spill over :

Volume of the stationary liquid = Volume of the liquid in motion.

$$\Rightarrow \frac{\pi D^2}{4} x1.5 = \frac{\pi D^2}{4} x z_o + \frac{1}{2} . \frac{\pi D^2}{4} (z_1 - z_0) \Rightarrow 1.5 = z_0 + \frac{1}{2} . (2 - z_0) \Rightarrow z_0 = 1.0m$$

$$z_1 - z_0 = \frac{{\omega_{max}}^2}{2g} . r^2 \Rightarrow 2 - 1 = \frac{{\omega_{max}}^2}{19.62} . 0.5^2 \Rightarrow \omega_{max} = 8.86\, rad/s$$

b) For the water depth to be $Z_0 = 0$ above the container's axis :

$$\frac{\pi D^2}{4} X1.5 = \frac{\pi D^2}{4} x z_0 + \frac{1}{2} . \frac{\pi D^2}{4} (z_1 - z_0)$$

$$z_0 = 0 \Rightarrow z_1 = 3m$$

$$z_1 - z_0 = \frac{{\omega_{max}}^2}{2g} . r^2 \Rightarrow 3 = \frac{{\omega_{max}}^2}{19.62} x 0.5^2 \Rightarrow \omega = 15.34 rad/s$$

c) For ω= 6 rad/s :

$$z_1 - z_0 = \frac{{\omega_{max}}^2}{2g} . r^2$$

$$\omega = 6\, rad/s \Rightarrow z_1 = \frac{6^2}{19.62} x 0.5^2 = 0.46\, m$$

$$\frac{\pi D^2}{4} X1.5 = \frac{\pi D^2}{4} x z_0 + \frac{1}{2} . \frac{\pi D^2}{4} (z_1 - z_0)$$

$$1.5 = z_0 + \frac{1}{2} . (z_1 - z_0)$$

[1] & [2] $\Rightarrow z_0 = 1.27m \quad z_1 = 1.73m$

$$\Longrightarrow p_{eksen} = 1.27 x \gamma_{su} = 12.46\, kN/m^2$$

$$\Longrightarrow p_{cidar} = 1.73 x \gamma_{su} = 16.97\, kN/m^2$$