Unit system and dimensional homogeneity

Question 1:

The Specific weight of water is 1000 kg/m^3. Using this given value, find the specific mass of water in SI units (g=9.81 m/s^2).

Solution 1:

The specific mass of water in SI units:

$1\,\mathrm{kg_f} = 9.81\,\mathrm{N} \Rightarrow \gamma_{su} = 1000\,\mathrm{kg_f m^{-3}} = 9810\,\mathrm{Nm^{-3}} = \rho_{su} \cdot g \Rightarrow \rho_{su} = 9810/9.81 = 1000\,\mathrm{N\,s^2 m^{-4}}$

Question 2:

Write the dimensions of the physical quantities and units in SI system of the parameters given below.

Dimension	Unit	SI
Force		
Tensor		
Velocity		
Acceleration		
Moment		
Specific mass		
Specific Weight		
Kinematic viscosity		
Dynamic viscosity		
Work		
Power		

Solution 2:

Dimension	Unit	SI
Force	F	N
Stress	F L^{-2}	N m^{-2}
Velocity	L T^{-1}	m s^{-1}
Acceleration	L T^{-2}	m s^{-2}
Moment	F L	N m
Specific Mass	F T^2 L^{-4}	kg m^{-3}
Specific Weight	F L^{-3}	N m^{-3}
Kinematic	L^2 T^{-1}	m^2 s^{-1}
Dynamic viscosity	F T L^{-2}	N s m^{-2}
Work	F L	N m (Joule)
Power	F L T^{-1}	N m s^{-1} (Watt)

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Question 3:

Let's have an oil with a volume *V*=200 lt and weighs G=1785 N. Determine the mass, specific weight and specific mass of the oil.

Solution 3:

$1\ lt = 10^{-3} m^3 \ \rightarrow\ V = 0.2\ m^3$

$$G = m.g \ \rightarrow\ m = \frac{1785}{9.81} = 182\ kg$$

$$G = \gamma_{oil}.V \ \rightarrow\ \gamma_{oil} = \frac{1785}{0.2} = 8925\ N/m^3$$

$$\gamma_{oil} = \rho_{oil}.g \ \rightarrow\ \rho_{oil} = \frac{8925}{9.81} = 909.79\ kg/m^3$$

Question 4:

Find the specific masses and kinematic viscosities of the fluids with specific weights and dynamic viscosities given below.

$$\gamma_{Ether} = 7063\ \frac{N}{m^3} \qquad \mu_{Ether} = 228.6\ N.s/m^2$$

$$\gamma_{Mercury} = 132886\ \frac{N}{m^3} \qquad \mu_{Mercury} = 1560\ N.s/m^2$$

$$\gamma_{Glycerine} = 12360.6\ \frac{N}{m^3} \qquad \mu_{Glycerine} = 799515\ N.s/m^2$$

Solution 4:

$$\gamma_{Ether} = \rho_{Eter}.g \ \rightarrow\ \rho_{Ether} = \frac{7063}{9.81} = 719.98\ \frac{kg}{m^3} \qquad \mu_{Ether} = \rho_{Ether}.\vartheta_{Ether}$$

$$\rightarrow\ \vartheta_{Ether} = \frac{228.6}{719.98} = 0.32\ m^2/s$$

$$\gamma_{Mercury} = \rho_{Mercury}.g \ \rightarrow\ \rho_{Mercury} = \frac{132886}{9.81} = 13545.97\ \frac{kg}{m^3} \qquad \mu_{Mercury}$$

$$= \rho_{Mercury}.\vartheta_{Mercury} \ \rightarrow\ \vartheta_{Mercury} = \frac{1560}{13545.97} = 0.12\ m^2/s$$

$$\gamma_{Glycerine} = \rho_{Glycerine}.g \ \rightarrow\ \rho_{Glycerine} = \frac{12360.6}{9.81} = 1260\ \frac{kg}{m^3} \qquad \mu_{Glycerine}$$

$$= \rho_{Glycerine}.\vartheta_{Glycerine} \ \rightarrow\ \vartheta_{Glycerine} = \frac{799515}{1260} = 634.54\ m^2/s$$

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Question 5:

Find the following parameters of an object weighing 9810 N by considering the standard acceleration of gravity as g=9.81 m/s^2.

a) Mass

b) Weight of the object on the moon (g_{moon}=1.62 m/s^2)

c) The acceleration when a horizontal force of 3924 N is applied both on the earth and the moon.

Solution 5:

a) $G = m.g \quad \rightarrow \quad m = \frac{9810}{9.81} = 1000\ kg$

b) $G_{moon} = m.g_{moon} \rightarrow G_{moon} = 1000x1.62 = 1620\ kg$

c) $F = m.a \quad \rightarrow \quad a = \frac{3924}{1000} = 3.92\ m^2/s^2$

Question 6:

The volume of glycerin with 1200 kg mass is 0.952 m^3. Find the weight, specific mass and specific weight of the glycerin.

Solution 6:

In the SI system

$P = m \cdot g \quad \Rightarrow \quad P = 1200 \cdot 9.81 = 11772\ \mathrm{N}$

$m = \rho \cdot V \quad \Rightarrow \quad \rho = \frac{1200}{0.952} = 1260.50\ \mathrm{kg\, m^{-3}}$

$\gamma = \rho \cdot g = P/V \quad \Rightarrow \quad \gamma = 12365.55\ \mathrm{N\, m^{-3}}$

Question 7:

The equation of drag force acting on a very slowly moving spherical particle in a fluid is given as F=3.π.μ.D.V. In the equation, μ is the dynamic viscosity with a dimension [F T L^{-2}], and D and V are the diameter and the velocity of the particle respectively.

a) What is the dimension of the (3π) constant multiplier?

b) Is this equation dimensionally homogeneous?

Solution 7:

$F = 3.\pi.\mu.D.V$

$\mu = [F.L^{-2}.T]; \quad D = [L]; \quad V = [L.T^{-1}]$

$[F] = [3\pi].[F.L^{-2}.T][L].[L.T^{-1}]$

$$[3\pi] = \frac{[F]}{[F]} = non - dimensional$$

Since the coefficient 3π is non–dimensional, the equation satisfies dimensional homogeneity.

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Question 8:

Based on experiments done by Henry DARCY (1803-1858), the following equation was proposed for determining head loss of friction. $h_k = f\,\dfrac{L}{D}\dfrac{v^2}{2g}$, where

h_k: Energy loss

L: Pipe length

D: Pipe diameter

f: Darcy-Weisbach friction coefficient

V: Cross-sectional average velocity of the fluid

g: Acceleration of gravity

Show that DARCY Equation is fulfilled in terms of dimensional homogeneity.

Solution 8:

$$h_k = f\,\frac{L}{D}\frac{v^2}{2g}$$

$$h_k = [L] \quad f = [-] \quad L = [L] \quad D = [L] \quad V = \left[LT^{-1}\right] \quad g = \left[LT^{-2}\right]$$

$$[L] = [-]\frac{[L]}{[L]}\frac{\left[L^2T^{-2}\right]}{\left[LT^{-2}\right]} \quad \Rightarrow \quad [L] = [L]$$

The equation satisfies dimensional homogeneity.

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Question 9:

The equation of the discharge of flow over a spillway is given in British unit system as follows:

$$Q = 3.09\,B\,H^{3/2}$$

,where H is the height of water above the spillway crest [L]=ft, B is width of the spillway [L]=ft, Q is discharge over the spillway [L^3/s]=ft^3/s

a) Is the quantity 3.09 dimensionally homogeneous?

b) Can we use this equation with other unit systems? (Note: 1 ft=0.3048 m).

Solution 9:

$$Q = \left[L^3 T^{-1}\right]$$
$$B = [L]$$
$$H = [L]$$
$$\left[L^3 T^{-1}\right] = [3.09][L]\left[L^{3/2}\right] \quad \Rightarrow \quad [3.09] = \left[L^{1/2} T^{-1}\right]$$

The coefficient 3.09 has a $\left[L^{1/2} T^{-1}\right]$ dimension of the equation satisfies dimensional homogeneity.

If the equation needs to be used in another unit system (for example SI unit system), a change has to be made depending on the units of the coefficient.

$$1\,\text{ft} = 0.3048\,\text{m} \quad \Rightarrow \quad 3.09\,\text{ft}^{1/2}\text{s}^{-1} = 3.09\left(0.3048\,\text{m}\right)^{1/2}\text{s}^{-1} = 1.71\,\text{m}^{1/2}\text{s}^{-1}$$
$$\Rightarrow \quad Q = 1.71\,B\,H^{3/2}$$